Partitions, quantum group actions and rigidity
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Compact matrix quantum groups

- Compact (orthogonal) Lie group $G \subset O_N(\mathbb{R})$
  $u_{ij} : G \to \mathbb{R}, g \mapsto g_{ij}$ coordinate function, $u := (u_{ij}) \in \mathbb{M}_N(C(G))$.
  Group multiplication $\leftrightarrow \Delta(u_{ij})(g,h) := u_{ij}(gh) = \sum_k u_{ik}(g)u_{kj}(h)$.
- Woronowicz: (orthogonal) compact matrix quantum group
  $\mathbb{G} = (A, u)$ such that
  - $A = C^*(u_{ij} \mid i,j = 1,\ldots,N)$
  - $u \in \mathbb{M}_N(A)$ such that $u_{ij} = u_{ij}^*$, $uu^T = u^T u = \mathrm{id}$
  - $*$-homomorphism $\Delta : A \to A \otimes A, \Delta(u_{ij}) = \sum_k u_{ik} \otimes u_{kj}$

  Write $A = C(\mathbb{G})$.
- Example: S.Z. Wang's free orthogonal quantum group $\mathbb{G} = O_N^+$
  where $A =$ universal $C^*(u_{ij} = u_{ij}^* \mid i,j = 1,\ldots,N; uu^T = u^T u = \mathrm{id})$

Categorical approach via Tannaka-Krein

- $u^{\otimes k} := (u_{i_1 j_1} \cdots u_{i_k j_k}) \in \mathbb{M}_N^{\otimes k} \otimes A$ with intertwiners

$$\mathrm{Mor}_{\mathbb{G}}(k,l) := \{T : (\mathbb{C}^N)^{\otimes k} \to (\mathbb{C}^N)^{\otimes l} \mid T u^{\otimes k} = u^{\otimes l} T\},\ k,l \in \mathbb{N}$$

- A collection of vector spaces of operators

$$\mathcal{R}_{\mathbb{G}} := \cup_{k,l} \mathrm{Mor}_{\mathbb{G}}(k,l) \subset \cup_{k,l} B((\mathbb{C}^N)^{\otimes k}, (\mathbb{C}^N)^{\otimes l})$$

  - contains $\mathrm{id}_{\mathbb{C}^N}$ and $1 \mapsto \sum_k e_k \otimes e_k$
  - stable under $\circ, \otimes, *$

- Tannaka-Krein reconstruction (Woronowicz 88'):

$$\mathbb{G} \leftrightarrow \mathcal{R}_{\mathbb{G}} \quad \text{one-to-one correspondence}$$

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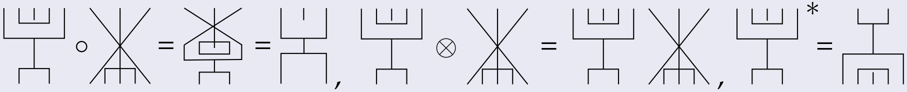
Category of partitions

A collection of partitions of two lines of points, such that:

- contains $|$ and $\sqcap$ - stable under $\circ, \otimes, *$



Partitions & quantum groups

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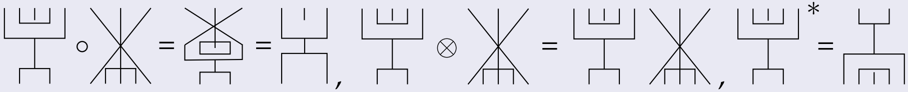

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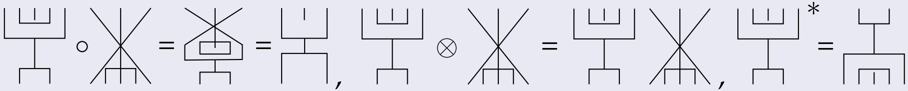

- Banica: $\mathcal{R}_{O_N^+} \leftrightarrow$ category of all noncrossing pair partitions
- Banica-Speicher:

any category of partitions $\rightarrow \mathcal{R}_{\mathbb{G}}$ for some $\mathbb{G}$

Such a $\mathbb{G}$ is called an "easy quantum group".

PARTITIONS & QUANTUM GROUPS

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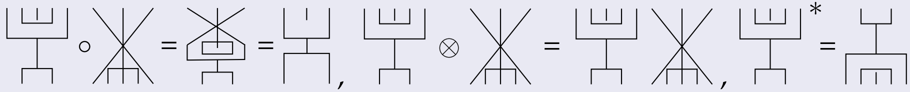

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GOAL for today: a dynamical version!

B unital C*-algebra

An action of $\mathbb{G}$ on B is an $*$-homomorphism $\alpha : B \to B \otimes C(\mathbb{G})$ with

- (coaction property) $(\alpha \otimes \mathrm{id}) \circ \alpha = (\mathrm{id} \otimes \Delta) \circ \alpha$
- (nondegeneracy condition) $\overline{\mathrm{span}}\{(1 \otimes C(G))\alpha(B)\} = B \otimes C(\mathbb{G})$

α is called ergodic if the fixed point space

$$B^{\mathbb{G}} := \{b \in B \mid \alpha(b) = b \otimes 1\} = \mathbb{C}1.$$

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Dual viewpoint: consider an **ergodic** α for the sequel.

Recall $u^{\otimes k} : (\mathbb{C}^N)^{\otimes k} \to (\mathbb{C}^N)^{\otimes k} \otimes C(\mathbb{G})$ & $\alpha : B \to B \otimes C(\mathbb{G})$

$\Rightarrow \mathbb{G} \curvearrowright (\mathbb{C}^N)^{\otimes k} \otimes B$ by

$$(u^{\otimes k})_{(13)}\alpha_{(23)} : (\mathbb{C}^N)^{\otimes k} \otimes B \to (\mathbb{C}^N)^{\otimes k} \otimes B \otimes C(\mathbb{G}).$$

ACTIONS OF COMPACT MATRIX QUANTUM GROUPS

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$$(u^{\otimes k})_{(13)}\alpha_{(23)} : (\mathbb{C}^N)^{\otimes k} \otimes B \to (\mathbb{C}^N)^{\otimes k} \otimes B \otimes C(\mathbb{G}).$$

- The fixed point space $K_k := ((\mathbb{C}^N)^{\otimes k} \otimes B)^{\mathbb{G}}$ is a Hilbert space.
- $K_k = (\mathbb{C}^N)^{\otimes k}$ if $B = C(\mathbb{G})$ and $\alpha = \Delta$; $K_k \otimes K_l \subset K_{k+l}$.
- $T \in \mathrm{Mor}_{\mathbb{G}}(k, l) \Rightarrow T \otimes \mathrm{id} \in B(K_k, K_l)$.

Tannaka-Krein reconstruction for actions

Recall Tannaka-Krein reconstruction for quantum groups
$\Rightarrow$ recognize $\mathbb{G}$ (in particular the action $\Delta : C(\mathbb{G}) \to C(\mathbb{G}) \otimes C(\mathbb{G})$) out of categorical data:

- $u^{\otimes k} \otimes u^{\otimes l} \cong u^{\otimes k+l} \to$ Hilbert spaces $((\mathbb{C}^N)^{\otimes k})_{k \in \mathbb{N}}$ with unitary $(\mathbb{C}^N)^{\otimes k} \otimes (\mathbb{C}^N)^{\otimes l} \cong (\mathbb{C}^N)^{\otimes k+l}$
- $T \in \mathrm{Mor}_{\mathbb{G}}(k, l) \to T \in B((\mathbb{C}^N)^{\otimes k}, (\mathbb{C}^N)^{\otimes l})$, compatible with $\otimes, \circ, *$.

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- $T \in \mathrm{Mor}_{\mathbb{G}}(k,l) \to T \in B((\mathbb{C}^N)^{\otimes k}, (\mathbb{C}^N)^{\otimes l})$, compatible with $\otimes, \circ, *$.

Theorem (Pinzari-Roberts 08', Neshveyev 14', Freslon-Taipe-W.)

Assume that we have

- *Hilbert spaces $(K_k)_{k\in\mathbb{N}}$ with isometric inclusions $\iota : K_k \otimes K_l \subset K_{k+l}$*
- *$T \in \mathrm{Mor}_{\mathbb{G}}(k,l) \to \varphi(T) \in B(K_k, K_l)$, compatible with $\otimes, \circ, *, \iota$.*

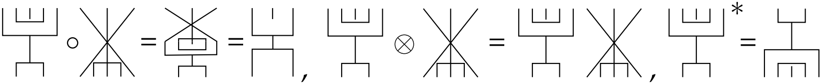
Then we may construct $\alpha : B \to B \otimes C(\mathbb{G})$ (with $K_k = ((\mathbb{C}^N)^{\otimes k} \otimes B)^{\mathbb{G}}$).

Main results: find such $(K_k)_{k\in\mathbb{N}}$ and φ out of partitions.

Partitions & quantum actions

Recall Banica-Speicher's approach:

- Partitions with operations $\circ, \otimes, *$



- A category of partitions $\mathcal{C} \to$ easy quantum group $\mathbb{G} = \mathbb{G}_N(\mathcal{C})$

  $\mathcal{C}(k,l) :=$ partitions of k upper points and l lower points in $\mathcal{C}$

Example ("module of projective partitions")

If $\mathcal{P}$ is a subset of projective partitions ($p = p^* = p^2$) such that

- (write $\mathcal{P}_k = \mathcal{P}(k,k)$) $\mathcal{P}_k \otimes \mathcal{P}_l \subset \mathcal{P}_{k+l}$;
- $r \in \mathcal{C}(k,l), p \in \mathcal{P}_k \Rightarrow rpr^* \in \mathcal{P}_l$.

$\Rightarrow$ $(\mathcal{P}_k)_{k\in\mathbb{N}}$ has a pre-inner product, compatible with $\varphi(r) : p \mapsto rpr^*$
$\Rightarrow$ an action of $\mathbb{G}_N(\mathcal{C})$.

Example ("module of projective partitions over $\mathcal{C}$")

If $\mathcal{P}$ is a subset of projective partitions ($p = p^* = p^2$) such that

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- $r \in \mathcal{C}(k,l), p \in \mathcal{P}_k \Rightarrow rpr^* \in \mathcal{P}_l$.

$\Rightarrow (\mathcal{P}_k)_{k\in\mathbb{N}}$ has a pre-inner product, and $\varphi(r) : p \mapsto rpr^*$
$\Rightarrow$ an action α of $\mathbb{G}_N(\mathcal{C})$.

Remarks

- The assumption is satisfied if $\mathcal{P}$ = all projective partitions in $\mathcal{C}$. In particular, for $\mathcal{C}$ = all noncrossing pair partitions ($\mathbb{G}_N(\mathcal{C}) = O_N^+$)

$$O_N^+ \curvearrowright C^*(x_i = x_i^* \mid \sum_i x_i^2 = 1, i = 1, \dots, N), \quad \alpha(x_i) = \sum_k x_k \otimes u_{ki}.$$

- $\mathcal{C}' \subset \mathcal{C} \Rightarrow \mathcal{P}$ module over $\mathcal{C}' \Rightarrow$ induced action $\mathrm{Ind}_{\mathbb{G}_N(\mathcal{C})}^{\mathbb{G}_N(\mathcal{C}')}(\alpha)$.

Example (module of line partitions)

For $\mathcal{L} = \cup_k \mathcal{L}_k := \cup_k \mathcal{C}(0,k)$ and $\mathcal{C}' \subset \mathcal{C}$, we have

- $\mathcal{L}_k \otimes \mathcal{L}_l \subset \mathcal{L}_{k+l}$;
- $r \in \mathcal{C}'(k,l), p \in \mathcal{L}_k \Rightarrow rp \in \mathcal{L}_l$.

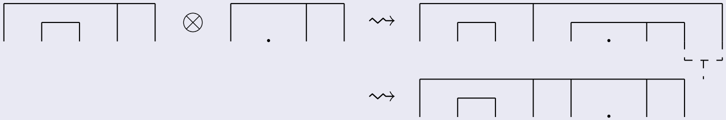
$\Rightarrow \alpha : \mathbb{G}_N(\mathcal{C}') \curvearrowright C(\mathbb{G}_N(\mathcal{C}) \backslash \mathbb{G}_N(\mathcal{C}'))$ translation action.

PARTITIONS & QUANTUM ACTIONS

Example (1-shifted line partitions)

For $\mathcal{C} \supset$ {noncrossing partitions}, $\mathcal{C}' \subset \mathcal{C}$ and $\mathcal{L} = \cup_k \mathcal{L}_k := \cup_k \mathcal{C}(0, k+1)$,

- $\mathcal{L}_k \otimes \mathcal{L}_l \subset \mathcal{L}_{k+l}$



- $r \in \mathcal{C}'(k,l), p \in \mathcal{L}_k \Rightarrow rp \in \mathcal{L}_l$.

$\Rightarrow$ For $\mathcal{C}' = \mathcal{C}$, $\alpha : \mathbb{G}_N(\mathcal{C}) \curvearrowright \mathbb{C}^N$, $\alpha(e_i) = \sum_k e_k \otimes u_{ki}$ permutation action;

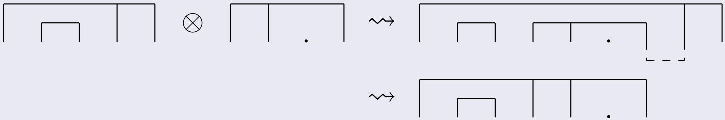
For $\mathcal{C}' \subset \mathcal{C}$, induced action $\mathrm{Ind}_{\mathbb{G}_N(\mathcal{C})}^{\mathbb{G}_N(\mathcal{C}')}(\alpha)$.

Partitions & quantum actions

Example (2-shifted line partitions)

For $\mathcal{C}' \subset \mathcal{C}$ and $\mathcal{L} = \cup_k \mathcal{L}_k := \cup_k \mathcal{C}(0, k+2)$,

- $\mathcal{L}_k \otimes \mathcal{L}_l \subset \mathcal{L}_{k+l}$



- $r \in \mathcal{C}'(k,l), p \in \mathcal{L}_k \Rightarrow rp \in \mathcal{L}_l$.

$\Rightarrow$ For $\mathcal{C}' = \mathcal{C}$, $\alpha : \mathbb{G}_N(\mathcal{C}) \curvearrowright \mathbb{M}_N(\mathbb{C}), \alpha(M) = u^*(M \otimes \mathrm{id})u$;
For $\mathcal{C}' \subset \mathcal{C}$, induced action $\mathrm{Ind}_{\mathbb{G}_N(\mathcal{C})}^{\mathbb{G}_N(\mathcal{C}')}(\alpha)$.

- Yetter-Drinfeld structure: (imprecise idea)
  $\mathbb{G} \curvearrowright^{\alpha} B \curvearrowleft^{\triangleleft} \hat{\mathbb{G}}$ "compatibly"
  For example, $B = C(\mathbb{G})$, $\alpha = \Delta : C(\mathbb{G}) \to C(\mathbb{G}) \otimes C(\mathbb{G})$,
  $\triangleleft = \mathrm{ad} : C(\mathbb{G}) \otimes C(\mathbb{G}) \to C(\mathbb{G})$, $a \triangleleft b = S(b_{(1)})ab_{(2)}$
- Freslon-Taipe-W.: Tannaka-Krein for Yetter-Drinfeld structures

$$\mathrm{a}_k : ((\mathbb{C}^N)^{\otimes k} \otimes B \supset) K_k \to K_{k+2}, \quad \sum_r f_r \otimes x_r \mapsto \sum_{i,j,r} e_i \otimes f_r \otimes e_j \otimes (x_r \triangleleft u_{ji})$$

Categorical "compatibility" of $(\mathrm{a}_k)_{k\in\mathbb{N}} \leftrightarrow$ Yetter-Drinfeld condition

Then an simple combinatorial analysis yields:

Proposition

The action $O_N^+ \curvearrowright C^*(x_i = x_i^* \mid \sum_i x_i^2 = 1, 1 \le i \le N), \alpha(x_i) = \sum_k x_k \otimes u_{ki}$ does not admit any Yetter-Drinfeld structure.

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The action $\mathbb{G}_N(\{noncrossing\ partitions\}) = S_N^+ \curvearrowright \mathbb{C}^N, \alpha(e_i) = \sum_k e_k \otimes u_{ki}$ does not admit any Yetter-Drinfeld structure.

Partitions & rigidity of actions

- Goswami 11′:
  - "there has not been any example of faithful action of a genuine compact quantum group on $C(X)$ when X is connected."
  - "conjecture that the quantum permutations are the only possible actions of genuine compact quantum groups on classical spaces".
- Huang 13′:
  - construct faithful non-ergodic $S_N^+ \curvearrowright$ compact connected X
  - reformulated question: faithful ergodic genuine quantum group actions on compact connected spaces?
- Goswami-Joardar GAFA 18′, Goswami Adv 20′:
  no genuine quantum actions on compact connected smooth manifolds.

Theorem (Freslon-Taipe-W.)

Free easy quantum groups ($\mathbb{G}_N(\mathcal{C})$ for noncrossing $\mathcal{C}$, e.g., $O_N^+, S_N^+, H_N^+, \ldots$) cannot act ergodically on a compact connected X unless X is a point.

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- multiplication in $C(X)$ $\rightleftarrows$ decomposition into irreducible components

$$(H_u \otimes C(X))^{\mathbb{G}} \otimes (H_{u'} \otimes C(X))^{\mathbb{G}} \subset \oplus_v (H_v \otimes C(X))^{\mathbb{G}}$$

- commutativity of $C(X)$ $\rightleftarrows$ symmetries in the decomposition

PARTITIONS & RIGIDITY OF ACTIONS

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- enforce a nontrivial finite-dimensional C*-subalgebra of $C(X)$.

Thank you very much!